

1YGB - MPI PART L - QUESTION 1

a) SOLVING THE LINEAR INEQUALITY

$$\Rightarrow \frac{x+2}{3} < 3x-1$$

$$\Rightarrow x+2 < 3(3x-1)$$

$$\Rightarrow x+2 < 9x-3$$

$$\Rightarrow -8x < -5$$

$$\Rightarrow \underline{x > \frac{5}{8}}$$

b) SOLVING THE QUADRATIC INEQUALITY

$$\Rightarrow x+6(x^2+2) > 20$$

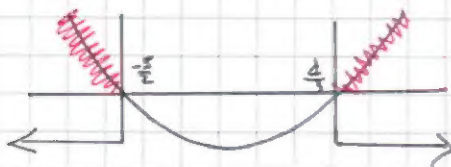
$$\Rightarrow x+6x^2+6x > 20$$

$$\Rightarrow 6x^2+7x-20 > 0$$

FACTORIZE OR USE THE QUADRATIC FORMULA TO FIND THE CRITICAL VALUES

$$\Rightarrow (3x-4)(2x+5) > 0$$

$$\Rightarrow \text{C.V.} = \begin{matrix} \frac{4}{3} \\ -\frac{5}{2} \end{matrix}$$



$$\underline{x < -\frac{5}{2} \quad \text{OR} \quad x > \frac{4}{3}}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 6 \times (-20)}}{2 \times 6}$$

$$x = \frac{-7 \pm \sqrt{529}}{12}$$

$$x = \frac{-7 \pm 23}{12}$$

$$x = \begin{matrix} \frac{4}{3} \\ -\frac{5}{2} \end{matrix}$$

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1YGB - MPI PAPER L - QUESTION 2

USING THE STANDARD BINOMIAL EXPANSION FORMULA

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{n} a^0 b^n$$

$$\left(x + \frac{2}{x}\right)^4 = \binom{4}{0} (x)^4 \left(\frac{2}{x}\right)^0 + \binom{4}{1} (x)^3 \left(\frac{2}{x}\right)^1 + \binom{4}{2} (x)^2 \left(\frac{2}{x}\right)^2 + \binom{4}{3} (x)^1 \left(\frac{2}{x}\right)^3 + \binom{4}{4} (x)^0 \left(\frac{2}{x}\right)^4$$

$$\left(x + \frac{2}{x}\right)^4 = (1 \times x^4 \times 1) + (4 \times x^3 \times \frac{2}{x}) + (6 \times x^2 \times \frac{4}{x^2}) + (4 \times x \times \frac{8}{x^3}) + (1 \times 1 \times \frac{16}{x^4})$$

$$\left(x + \frac{2}{x}\right)^4 = x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$$

1XGB - MPI PART L - QUESTION 3

TIDY AND CREATE $\tan y$

$$\Rightarrow 2\sin y + 5\cos y = 2\cos y$$

$$\Rightarrow 2\sin y = -3\cos y$$

$$\Rightarrow \frac{2\sin y}{\cos y} = \frac{-3\cos y}{\cos y}$$

$$\Rightarrow 2\tan y = -3$$

$$\Rightarrow \tan y = -\frac{3}{2}$$

$$\underline{\arctan\left(-\frac{3}{2}\right) = -56.3^\circ}$$

$$\Rightarrow y = -56.3^\circ \pm 180n \quad n=0,1,2,3,\dots$$

$$\therefore \underline{\underline{y_1 = 123.7^\circ}}$$

$$\underline{\underline{y_2 = 303.7^\circ}}$$

YGB-MPI PAPER L - QUESTION 4

LET $y = f(x) = x^3 - 4x + 1$

THEN $f(x+h) = (x+h)^3 - 4(x+h) + 1$

$$= (x+h)(x+h)^2 - 4x - 4h + 1$$

$$= (x+h)(x^2 + 2xh + h^2) - 4x - 4h + 1$$

$$= x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 - 4x - 4h + 1$$

$$= x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h + 1$$

BY THE FORMAL DEFINITION OF THE DERIVATIVE WE HAVE

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{(x^3 + 3x^2h + 3xh^2 + h^3 - 4x - 4h + 1) - (x^3 - 4x + 1)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{4x} - 4h + \cancel{1} - \cancel{x^3} + \cancel{4x} - \cancel{1}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{3x^2h + 3xh^2 + h^3 - 4h}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[3x^2 + 3xh + h^2 - 4 \right]$$

$$= \underline{3x^2 - 4}$$

✓ DERIVADO

1YGB - MPI PAPER 1 - QUESTION 5

BY THE COSINE RULE - "BACKWARDS"

$$\Rightarrow |AB|^2 = |AC|^2 + |BC|^2 - 2|AC||BC|\cos\theta$$

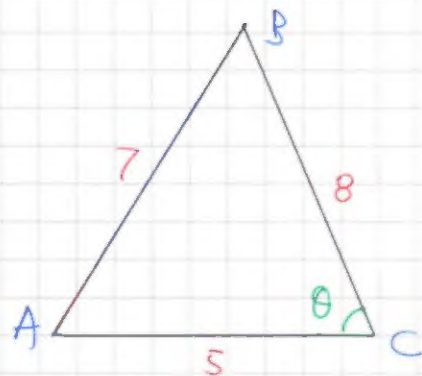
$$\Rightarrow 7^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \times \cos\theta$$

$$\Rightarrow 80\cos\theta = 25 + 64 - 49$$

$$\Rightarrow 80\cos\theta = 40$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = 60^\circ$$



USING THE TRIGONOMETRIC FORM FOR THE AREA OF THE TRIANGLE

$$\text{Area} = \frac{1}{2}|AC||BC|\sin\theta$$

$$\text{Area} = \frac{1}{2} \times 5 \times 8 \times \sin 60^\circ$$

$$\text{Area} = 20 \times \frac{\sqrt{3}}{2}$$

$$\text{Area} = 10\sqrt{3}$$

1YGB-MPI PAPER L - QUESTION 6

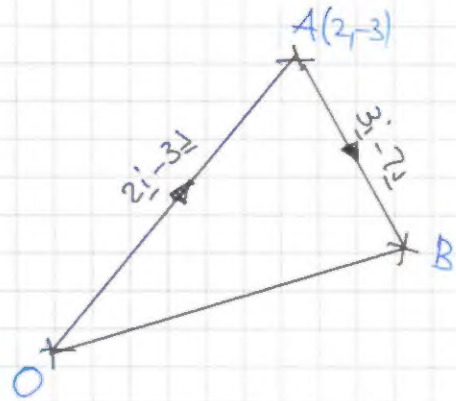
LOOKING AT A DIAGRAM

$$\Rightarrow \vec{OB} = \vec{OA} + \vec{AB}$$

$$\Rightarrow \vec{OB} = (2\hat{i} - 3\hat{j}) + (3\hat{i} - 7\hat{j})$$

$$\Rightarrow \vec{OB} = 5\hat{i} - 10\hat{j}$$

$$\therefore B(5, -10)$$



DISTANCE OF B FROM O IS GIVEN BY

$$|\vec{OB}| = |5\hat{i} - 10\hat{j}| = \sqrt{5^2 + (-10)^2} = \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}$$

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YGB - MPI PAPER L - QUESTION 7

a) Let $h(x) = \frac{1}{x}$

• $f(x) = h(x-2) = \frac{1}{x-2}$

↳ TRANSLATION, TO THE "RIGHT", BY 2 UNITS

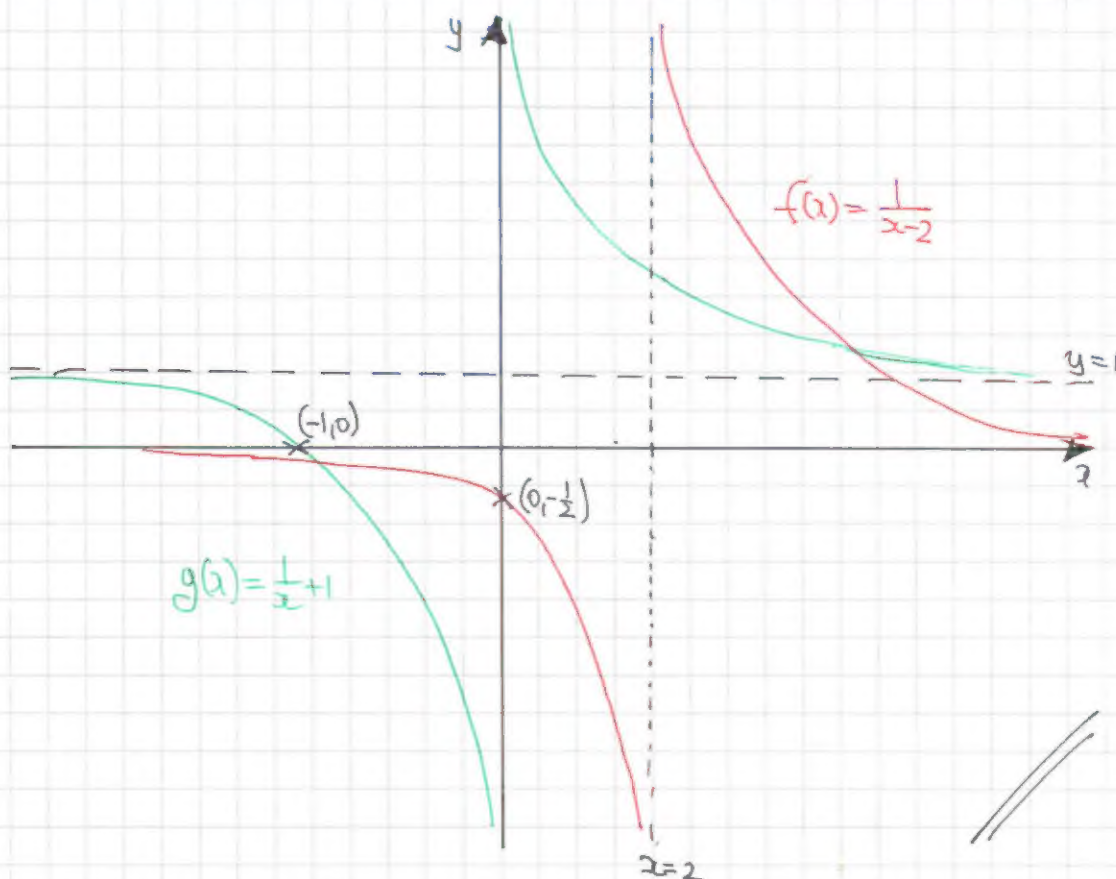
↳ TRANSLATION BY THE VECTOR $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ //

• $g(x) = h(x)+1 = \frac{1}{x}+1$

↳ TRANSLATION, "UPWARDS", BY 1 UNIT

↳ TRANSLATION BY THE VECTOR $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ //

b) SKETCHING USING THE TRANSFORMATION DESCRIPTIONS FROM PART (a)



c) SOLVING THEIR EQUATIONS SIMULTANEOUSLY

$$\left. \begin{array}{l} f(x) = y = \frac{1}{x-2} \\ g(x) = y = \frac{1}{x} + 1 \end{array} \right\} \Rightarrow \frac{1}{x-2} = \frac{1}{x} + 1$$

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IYGB - MPI PAPER L - QUESTION 7

$$\Rightarrow \frac{1}{x-2} = \frac{1}{2} + 1$$

$$\Rightarrow \frac{x}{x-2} = 1 + x$$

$$\Rightarrow x = 1(x-2) + x(x-2)$$

$$\Rightarrow x = x - 2 + x^2 - 2x$$

$$\Rightarrow 0 = x^2 - 2x - 2$$

MULTIPLY THROUGH BY 2

MULTIPLY THROUGH BY (x-2)

BY COMPLETING THE SQUARE

$$\Rightarrow (x-1)^2 - 1 - 2 = 0$$

$$\Rightarrow (x-1)^2 = 3$$

$$\Rightarrow x-1 = \begin{cases} \sqrt{3} \\ -\sqrt{3} \end{cases}$$

$$\Rightarrow x = \begin{cases} 1+\sqrt{3} \\ 1-\sqrt{3} \end{cases}$$

$$y = \frac{1}{x-2} = \begin{cases} \frac{1}{1+\sqrt{3}-2} = \frac{1}{\sqrt{3}-1} = \frac{1}{2}(1+\sqrt{3}) \\ \frac{1}{1-\sqrt{3}-2} = \frac{1}{-\sqrt{3}-1} = \frac{1}{2}(1-\sqrt{3}) \end{cases}$$

$$\therefore \underline{\left[1+\sqrt{3}, \frac{1}{2}(1+\sqrt{3})\right] \text{ \& } \left[1-\sqrt{3}, \frac{1}{2}(1-\sqrt{3})\right]}$$

1YGB - MPI PAPER L - QUESTION 8

a) BY THE FACTOR THEOREM

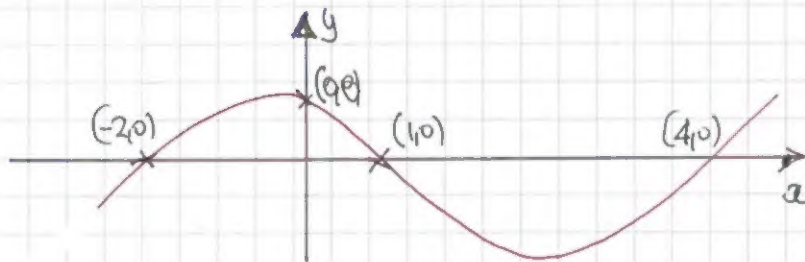
$$\begin{aligned} f(1) &= 1^3 - 3 \times 1^2 - 6 \times 1 + 8 \\ &= 1 - 3 - 6 + 8 \\ &= 0 \end{aligned}$$

INDEED A FACTOR

b) BY LONG DIVISION OR MANIPULATION

$$\begin{aligned} x^3 - 3x^2 - 6x + 8 &= x^2(x-1) - 2x(x-1) - 8(x-1) \\ &= (x-1)(x^2 - 2x - 8) \\ &= (x-1)(x+2)(x-4) \end{aligned}$$

c)



$$\begin{aligned} +x^3 &\Rightarrow \text{wavy line} \\ x=0 &\Rightarrow y=8 \quad (0,8) \\ y=0 &\Rightarrow x = \begin{matrix} -2 \\ 1 \\ 4 \end{matrix} \quad (-2,0), (1,0), (4,0) \end{aligned}$$

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d) SOLVING SIMULTANEOUSLY

$$\begin{cases} y = x^4 + x^3 - 4x^2 - 10 \\ y = x^4 - x^3 + 2x^2 + 12x - 26 \end{cases} \Rightarrow$$

$$\Rightarrow \cancel{x^4} + x^3 - 4x^2 - 10 = \cancel{x^4} - x^3 + 2x^2 + 12x - 26$$

$$\Rightarrow 2x^3 - 6x^2 - 12x + 16 = 0$$

$$\Rightarrow x^3 - 3x^2 - 6x + 8 = 0$$

USING PART (b) & $y = x^4 + x^3 - 4x^2 - 10$

$$x = \begin{matrix} 1 \\ -2 \\ 4 \end{matrix} \quad y = \begin{matrix} -12 \\ -18 \\ 246 \end{matrix}$$

$$\therefore \underline{P(-2, -18) \quad Q(1, -12) \quad R(4, 246)}$$

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1YGB - MPI PAGE 1 - QUESTION 9

MANIPULATE AS QUADS

$$\Rightarrow \ln x^2 + \frac{3}{\ln x} = 7$$

$$\Rightarrow 2\ln x + \frac{3}{\ln x} = 7$$

$$\Rightarrow 2(\ln x)^2 + 3 = 7\ln x$$

$$\Rightarrow 2(\ln x)^2 - 7\ln x + 3 = 0$$

FACTORIZE THE QUADRATIC

$$\Rightarrow (2\ln x - 1)(\ln x - 3)$$

$$\Rightarrow \ln x = \begin{matrix} \frac{1}{2} \\ 3 \end{matrix}$$

$$\Rightarrow a = \begin{matrix} e^{\frac{1}{2}} = \sqrt{e} \\ e^3 \end{matrix} //$$

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1YGB - MP1 PAPER 2 L - QUESTION 10

START BY FINDING THE CO-ORDINATES OF M

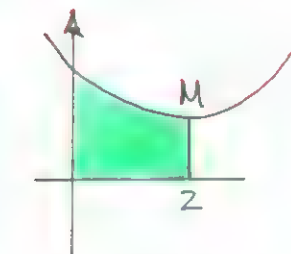
$$\Rightarrow y = x^2 - 4x + 8$$

$$\Rightarrow y = (x-2)^2 - 4 + 8$$

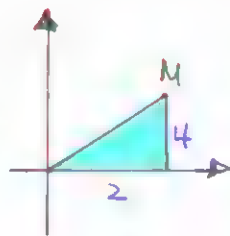
$$\Rightarrow y = (x-2)^2 + 4$$

$$\therefore M(2, 4)$$

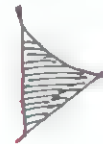
LOOKING AT THE PICTORIAL EQUATION BELOW



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$$\int_0^2 x^2 - 4x + 8 \, dx$$

$$\frac{1}{2} \times 2 \times 4 = 4$$

$$= \left[\frac{1}{3}x^3 - 2x^2 + 8x \right]_0^2$$

$$= \left(\frac{8}{3} - 8 + 16 \right) - (0)$$

$$= \frac{32}{3}$$

$$\underline{\text{Required Area}} = \frac{32}{3} - 4 = \frac{20}{3}$$

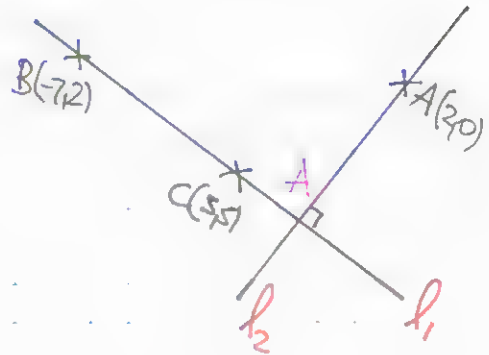
LYGB - MPI PAPER L - QUESTION 11

START BY A DIAGRAM

$$\begin{aligned} \bullet \text{ grad } BC &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 5}{-7 - 5} \\ &= \frac{-3}{-12} = \frac{1}{4} \end{aligned}$$

$$\bullet \text{ grad } l_1 = \frac{1}{4}$$

$$\bullet \text{ grad } l_2 = -4$$



FIND THE EQUATION OF l_1 , $m = \frac{1}{4}$, $C(5, 5)$ & l_2 , $m = -4$, $A(2, 0)$

$$\begin{aligned} \bullet l_1 : y - y_0 &= m(x - x_0) \\ y - 5 &= \frac{1}{4}(x - 5) \\ 4y - 20 &= x - 5 \\ 4y &= x + 15 \end{aligned}$$

$$\begin{aligned} \bullet l_2 : y - y_0 &= m(x - x_0) \\ y - 0 &= -4(x - 2) \\ y &= -4x + 8 \end{aligned}$$

SOLVING SIMULTANEOUSLY

$$\Rightarrow 4(-4x + 8) = x + 15$$

$$\Rightarrow -16x + 32 = x + 15$$

$$\Rightarrow -17x = -17$$

$$\Rightarrow x = 1$$

$$\begin{aligned} \text{q } y &= -4 \times 1 + 8 \\ y &= 4 \end{aligned}$$

$$\therefore \underline{D(1, 4)}$$

1YGB - MPI PAPER 1 - QUESTION 12

CONSIDER THE EXPANSION OF $(\sqrt{k}-1)^2$

$$\Rightarrow (\sqrt{k}-1)^2 \geq 0$$

$$\Rightarrow (\sqrt{k})^2 - 2 \times 1 \times \sqrt{k} + 1^2 \geq 0$$

$$\Rightarrow k - 2\sqrt{k} + 1 \geq 0$$

$$\Rightarrow k + 1 \geq 2\sqrt{k}$$

AS $\sqrt{k} > 0$ WE MAY DIVIDE IT

$$\Rightarrow \frac{k+1}{\sqrt{k}} \geq 2$$

AS REQUIRED

ALTERNATIVE BY DIFFERENTIATION

FIRSTLY LET US NOTE THAT AS k GETS LARGER, THE WHOLE EXPRESSION GETS LARGER WITHOUT BOUND, SO ANY STATIONARY POINT WILL BE AN ABSOLUTE MINIMUM

$$\text{e.g. } \lim_{k \rightarrow \infty} \left(\frac{k+1}{\sqrt{k}} \right) = \lim_{k \rightarrow \infty} \left(\sqrt{k} + \frac{1}{\sqrt{k}} \right)$$

$$y = \frac{k+1}{\sqrt{k}} = \frac{k}{k^{\frac{1}{2}}} + \frac{1}{k^{\frac{1}{2}}} = k^{\frac{1}{2}} + k^{-\frac{1}{2}}$$

$$\frac{dy}{dk} = \frac{1}{2}k^{-\frac{1}{2}} - \frac{1}{2}k^{-\frac{3}{2}}$$

SOLVING FOR ZERO, TO LOOK FOR MINIMUM

$$0 = \frac{1}{2}k^{-\frac{1}{2}} - \frac{1}{2}k^{-\frac{3}{2}}$$

YGB - MPI PAPER L - QUESTION 12

$$\Rightarrow \frac{1}{2}k^{-\frac{1}{2}} = \frac{1}{2}k^{-\frac{3}{2}}$$

$$\Rightarrow k^{-\frac{1}{2}} = k^{-\frac{3}{2}}$$

$$\Rightarrow \frac{1}{k^{\frac{1}{2}}} = \frac{1}{k^{\frac{3}{2}}}$$

$$\Rightarrow \frac{k^{\frac{3}{2}}}{k^{\frac{1}{2}}} = 1$$

As $k > 0$, we may divide

$$\Rightarrow k = 1$$

$$\therefore \left(\frac{k+1}{\sqrt{k}} \right)_{\min} = \frac{1+1}{\sqrt{1}} = \frac{2}{1} = 2$$

// AS REQUIRED

YGB - MPI PAPER L - QUESTION 13

START BY FINDING THE COORDINATES OF Q

$$y = x^2 - 14x + 48$$

$$y = (x-8)(x-6)$$

$$\therefore \underline{P(6,0)} \quad \underline{Q(8,0)}$$

FIND THE GRADIENT AT Q

$$\frac{dy}{dx} = 2x - 14$$

$$\left. \frac{dy}{dx} \right|_{x=8} = 2 \times 8 - 14 = \underline{2}$$

EQUATION OF TANGENT (L₁)

$$y - y_0 = m(x - x_0)$$

$$y - 0 = 2(x - 8)$$

$$\underline{y = 2x - 16}$$

AS L₂ IS PERPENDICULAR TO L₁ ITS GRADIENT IS $-\frac{1}{2}$

$$\frac{dy}{dx} = -\frac{1}{2} \Rightarrow 2x - 14 = -\frac{1}{2}$$

$$\Rightarrow 4x - 28 = -1$$

$$\Rightarrow 4x = 27$$

$$\Rightarrow x = \frac{27}{4} \leftarrow \text{POINT R}$$

$$y = \left(\frac{27}{4}\right)^2 - 14 \times \frac{27}{4} + 48$$

$$y = -\frac{15}{16}$$

$$\therefore \underline{R\left(\frac{27}{4}, -\frac{15}{16}\right)}$$

FIND THE EQUATION OF L₂

$$y - y_0 = m(x - x_0)$$

$$y + \frac{15}{16} = -\frac{1}{2}\left(x - \frac{27}{4}\right)$$

YGB - MPI PAPER L - QUESTION 13

SOLVING SIMULTANEOUSLY

$$y + \frac{15}{16} = -\frac{1}{2}\left(x - \frac{27}{4}\right) \quad \& \quad y = 2x - 16$$

$$\begin{aligned} &\rightarrow (2x - 16) + \frac{15}{16} = -\frac{1}{2}\left(x - \frac{27}{4}\right) \quad \leftarrow \times 16 \\ &32x - 256 + 15 = -8\left(x - \frac{27}{4}\right) \end{aligned}$$

$$32x - 241 = -8x + 54$$

$$40x = 295$$

$$x = \frac{59}{8}$$

$$\& \quad y = 2\left(\frac{59}{8}\right) - 16$$

$$y = -\frac{5}{4}$$

$$\therefore \underline{\underline{\left(\frac{59}{8}, -\frac{5}{4}\right)}}$$

1XGB - MPI PAPER L - QUESTION 14

a) COMPLETE THE SQUARE IN x & y

$$\Rightarrow x^2 + y^2 = 8y$$

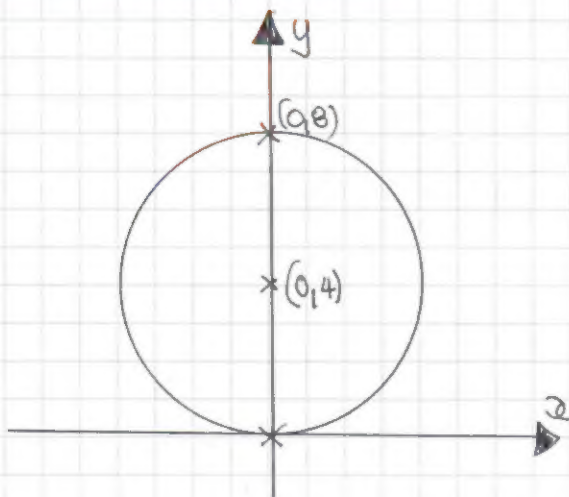
$$\Rightarrow x^2 + y^2 - 8y = 0$$

$$\Rightarrow x^2 + (y-4)^2 - 16 = 0$$

$$\Rightarrow x^2 + (y-4)^2 = 16$$

IE CENTRE AT $(0, 4)$ & RADIUS 4

b) SKETCHING THE CIRCLE



c) SOLVING SIMULTANEOUSLY

$$\left. \begin{array}{l} x^2 + y^2 = 8y \\ x + y = k \end{array} \right\} \Rightarrow \boxed{x = k - y}$$

$$\Rightarrow (k - y)^2 + y^2 = 8y$$

$$\Rightarrow k^2 - 2ky + y^2 + y^2 = 8y$$

$$\Rightarrow 2y^2 - 2ky - 8y + k^2 = 0$$

$$\Rightarrow 2y^2 - (2k + 8)y + k^2 = 0$$

1YGB - MPI PAPER L - QUESTION 14IF TANGENT WE MUST HAVE REPEATED ROOTS

$$b^2 - 4ac = 0 \Rightarrow [-(2k+8)]^2 - 4 \times 2 \times k^2 = 0$$

$$\Rightarrow (2k+8)^2 - 8k^2 = 0$$

$$\Rightarrow 4k^2 + 32k + 64 - 8k^2 = 0$$

$$\Rightarrow 0 = 4k^2 - 32k - 64$$

$$\Rightarrow \boxed{k^2 - 8k - 16 = 0}$$

$$\Rightarrow (k-4)^2 - 16 - 16 = 0$$

$$\Rightarrow (k-4)^2 = 32$$

$$\Rightarrow k-4 = \pm \sqrt{32}$$

$$\Rightarrow k = \begin{matrix} 4 + 4\sqrt{2} \\ 4 - \sqrt{2} \end{matrix} //$$

OR QUADRATIC FORMULA

$$k = \frac{8 \pm \sqrt{64 - 4 \times 1 \times (-16)}}{2 \times 1}$$

$$k = \frac{8 \pm \sqrt{64 + 64}}{2}$$

$$k = \frac{8 \pm \sqrt{128}}{2}$$

$$k = \frac{8 \pm 8\sqrt{2}}{2}$$

$$k = 4 \pm 4\sqrt{2}$$